HANDOUT 8

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1. Explain those terminologies: vector space, subspace, zero subspace, null space, column space, row space, linear transformation.

(Subspaces of $\mathbb{R}^n$ arise as the set of all solutions to a system of a homogeneous linear equations.)

2. Determine if the following spaces are vector spaces.

(1) The line $y = 2x$ in $\mathbb{R}^2$. Yes
(2) The solution set of a homogeneous system of linear equations. Yes
(3) The solution set of an inhomogeneous system of linear equations. No
(4) The kernel of a linear transformation. Yes
(5) The image of a linear transformation. No
(6) The span of a collection of vectors. Yes
(7) The set of all 2×2 invertible matrices. No
(8) The set of all 2×2 symmetric ($A = A^T$) matrices. Yes
(9) The set of all 2×2 skew-symmetric ($A = -A^T$) matrices. Yes

3. Show that the following set of vectors $\mathcal{B} = \{\mathbf{a}_1, \mathbf{a}_2, \mathbf{a}_3\}$ form a basis for $\mathbb{R}^3$, and find the coordinates of the vector u below in this coordinate-system (i.e. find $[u]_{\mathcal{B}}$).

$$\mathbf{a}_1 = \begin{bmatrix} 1 \\ -2 \\ -3 \end{bmatrix}, \mathbf{a}_2 = \begin{bmatrix} 1 \\ 6 \\ 5 \end{bmatrix}, \mathbf{a}_3 = \begin{bmatrix} 2 \\ 4 \\ 3 \end{bmatrix}, u = \begin{bmatrix} 5 \\ 6 \\ 1 \end{bmatrix}$$

Sol: $[u]_{\mathcal{B}} = \begin{bmatrix} 3 \\ 2 \\ 0 \end{bmatrix}$.