HANDOUT 7

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1. Let T be a linear transform with $T\begin{bmatrix}3\\1\end{bmatrix} = \begin{bmatrix}1\\2\end{bmatrix}, T\begin{bmatrix}2\\1\end{bmatrix} = \begin{bmatrix}5\\6\end{bmatrix}$. Determine the matrix A_T of T. (Hint: Find $\begin{bmatrix}1 & 5\\2 & 6\end{bmatrix}\begin{bmatrix}3 & 2\\1 & 1\end{bmatrix}^{-1}$.)

2. Explain those terminologies: subspace of $\mathbb{R}^n$, column space, null space, basis, standard basis, dimension of a nonzero subspace, rank.

Subspace of $\mathbb{R}^n$ arise as the set of all linear combinations of certain specified vectors.

Q: is $\mathbb{R}^1$ a subspace of $\mathbb{R}^2$?

3. The vector $\mathbf{x}$ is in a subspace H with a basis $\mathcal{B} = \{\mathbf{b}_1, \mathbf{b}_2\}$. Find the $\mathcal{B}$-coordinate vector of $\mathbf{x}$.

$$\mathbf{b}_1 = \begin{bmatrix} 2\\4\\2 \end{bmatrix}, \mathbf{b}_2 = \begin{bmatrix} 1\\-3\\5 \end{bmatrix}, \mathbf{x} = \begin{bmatrix} 3\\11\\-1 \end{bmatrix},$$

Sol: $\mathbf{x} = 2\mathbf{b}_1 - \mathbf{b}_2$, thus $[\mathbf{x}]_{\mathcal{B}} = \begin{bmatrix} 2\\-1 \end{bmatrix}$.

4. Given a matrix A, find bases for Col$A$ and Nul$A$, as well as the dimension of these subspaces.

$$A = \begin{bmatrix} 1 & 2 & 4 & -1\\ -1 & -3 & 2 & -6\\ 7 & 1 & 1 & 7\\ 5 & 2 & 4 & 3 \end{bmatrix}$$

Sol: do row reduction to

$$\begin{bmatrix} 1 & 2 & 4 & -1 & 0\\ -1 & -3 & 2 & -6 & 0\\ 7 & 1 & 1 & 7 & 0\\ 5 & 2 & 4 & 3 & 0 \end{bmatrix}$$

and get

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0\\ 0 & 1 & 0 & 0 & 0\\ 0 & 0 & 1 & 0 & 0\\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

Hence Col $A = \mathbb{R}^4$ and Nul A is zero space. The dimension of Col A is 4 and the dimension of Nul A is 0.