HANDOUT 3

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1. Explain the following terms: Linear combination, Span, vector equation, matrix equation, homogeneous linear system, Nonhomogeneous system.

2. How to determine if v is in the span of $v_1, \cdots, v_m$? If v is in the span of $v_1, \cdots, v_m$, how to find $a_1, \cdots, a_m$ such that $v = \sum_i a_i v_i$? (See example below.)

3. Determine if $\mathbf{b}$ is a linear combination of $\mathbf{a}_1, \mathbf{a}_2$ and $\mathbf{a}_3$. If so, write down the linear combination.

$$\mathbf{a}_1 = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \mathbf{a}_2 = \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix}, \mathbf{a}_3 = \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}, \mathbf{b} = \begin{bmatrix} -7 \\ 3 \\ -2 \end{bmatrix} \tag{1}$$

Sol: Yes, $\mathbf{b} = \mathbf{a}_1 - 4\mathbf{a}_2 + \mathbf{a}_3$

4. Solve the following system of equation: $\begin{bmatrix} 1 & 0 & 2 \\ -2 & 5 & 0 \\ 2 & 5 & 8 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 17 \\ 4 \\ 72 \end{bmatrix}$.

Sol:

$$\left\{ \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 17 \\ \frac{38}{5} \\ 0 \end{bmatrix} + s \begin{bmatrix} -2 \\ -\frac{4}{5} \\ 1 \end{bmatrix} : s \in \mathbb{R} \right\}$$

5. Find the parametric equation of the line through $\mathbf{a}$ parallel to $\mathbf{b}$.

$$\mathbf{a} = \begin{bmatrix} 7 \\ 2 \end{bmatrix}, \mathbf{b} = \begin{bmatrix} 3 \\ -4 \end{bmatrix}. \tag{2}$$

Sol:

$$\mathbf{x} = \begin{bmatrix} 7 \\ 2 \end{bmatrix} + t \begin{bmatrix} 3 \\ -4 \end{bmatrix}.$$

6. Find the parametric equation of the line through $\mathbf{a}$ and $\mathbf{b}$.

$$\mathbf{a} = \begin{bmatrix} 7 \\ 2 \end{bmatrix}, \mathbf{b} = \begin{bmatrix} 3 \\ -4 \end{bmatrix}. \tag{3}$$

Sol:

$$\mathbf{x} = (1-t) \begin{bmatrix} 7 \\ 2 \end{bmatrix} + t \begin{bmatrix} 3 \\ -4 \end{bmatrix}.$$

Takeaway:

- The parametric equation of the line through $\mathbf{a}$ parallel to $\mathbf{b}$: $\mathbf{x} = \mathbf{a} + t\mathbf{b}$.
- The parametric equation of the line through $\mathbf{a}$ and $\mathbf{b}$: $\mathbf{x} = t\mathbf{a} + (1-t)\mathbf{b}$.