HANDOUT 17

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The Gram-Schmidt Process.

1. Find a vector on $x + y + z = 0$ that is closest to $(1, 2, 3)$.

Hint: If you are using solutions to the least square problem, the x so that $|Ax - b|$ takes its minimal satisfies $A^T(Ax - b) = 0$.

2. Let $x_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$, $x_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$ and $x_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$. Compute

$$v_1 = x_1, \quad v_2 = x_2 - \frac{x_2 \cdot v_1}{v_1 \cdot v_1} v_1, \quad v_3 = x_3 - \frac{x_3 \cdot v_1}{v_1 \cdot v_1} v_1 - \frac{x_3 \cdot v_2}{v_2 \cdot v_2} v_2.$$

Check directly that $\{v_1, v_2, v_3\}$ are pairwise orthogonal

3. Find the QR factorization of

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}.$$

4. Find an orthogonal basis for the null space of

$$A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \end{bmatrix}$$