HANDOUT 15

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Orthogonal projection. The orthogonal decomposition theorem. Least-squares problem.

Exercise 1: Find the orthogonal projection of $\mathbf{x} = \begin{bmatrix} 3 \\ 2 \\ 5 \end{bmatrix}$ to the two dimensional subspace $W \subset \mathbb{R}^3$ spanned by $\mathbf{u}_1 = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$ and $\mathbf{u}_2 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$. (It is denoted as $\mathrm{proj}_W \mathbf{x}$.)

Exercise 2: Find the best approximation to $\mathbf{x}$ by vectors of the form $c_1\mathbf{v}_1 + c_2\mathbf{v}_2$.

$$\mathbf{v}_1 = \begin{bmatrix} 3 \\ -7 \\ 2 \\ 3 \end{bmatrix}, \mathbf{v}_2 = \begin{bmatrix} 2 \\ -1 \\ -3 \\ 1 \end{bmatrix}, \mathbf{x} = \begin{bmatrix} 1 \\ 1 \\ 0 \\ -1 \end{bmatrix}$$

Exercise 3: Find the distance of the point $\mathbf{x} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ to the two dimensional subspace $W \subset \mathbb{R}^3$ spanned by $\mathbf{u}_1 = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$ and $\mathbf{u}_2 = \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}$.

Exercise 4: Construct the normal equations for the following least-square problems $A\mathbf{x} = b$, and find a least-square solution $\mathbf{x}$.

$$A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 0 \end{bmatrix}, \quad b = \begin{bmatrix} 2 \\ 3 \\ 3 \end{bmatrix}$$

Exercise 5: Describe all least-squares solutions to the equation $A\mathbf{x} = b$.

$$A = \begin{bmatrix} 1 & 2 & 0 \\ 2 & 4 & 0 \\ 3 & 0 & 6 \\ 4 & 0 & 8 \end{bmatrix}, \quad b = \begin{bmatrix} 4 \\ 3 \\ 2 \\ 1 \end{bmatrix}$$