# HANDOUT 14

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- Inner product and distance

  **Exercise**: Compute the following quantities using these vectors:

$$\mathbf{u} = \begin{bmatrix} -1 \\ 3 \end{bmatrix}, \quad \mathbf{v} = \begin{bmatrix} 4 \\ 6 \end{bmatrix}, \quad \mathbf{w} = \begin{bmatrix} 3 \\ -4 \\ -5 \end{bmatrix}, \quad \mathbf{x} = \begin{bmatrix} 6 \\ -2 \\ 3 \end{bmatrix}.$$

  (1) $\mathbf{u} \cdot \mathbf{u}$, $\mathbf{v} \cdot \mathbf{u}$, $\frac{\mathbf{v} \cdot \mathbf{u}}{\mathbf{u} \cdot \mathbf{u}}$ and $\frac{\mathbf{u} \cdot \mathbf{v}}{\mathbf{v} \cdot \mathbf{v}}\mathbf{v}$
  (2) $\mathbf{w} \cdot \mathbf{w}$, $\mathbf{w} \cdot \mathbf{x}$, $\frac{\mathbf{x} \cdot \mathbf{w}}{\mathbf{w} \cdot \mathbf{w}}$ and $\frac{\mathbf{x} \cdot \mathbf{w}}{\mathbf{x} \cdot \mathbf{x}}\mathbf{x}$
  (3) $||\mathbf{u}||$, $||\mathbf{w}||$

  **Exercise**: Find the distances between the following pairs of vectors.

  (1)

$$\mathbf{x} = \begin{bmatrix} 2 \\ -3 \end{bmatrix}, \quad \mathbf{y} = \begin{bmatrix} -1 \\ -7 \end{bmatrix}$$

  (2)

$$\mathbf{x} = \begin{bmatrix} 0 \\ -1 \\ 2 \end{bmatrix}, \quad \mathbf{y} = \begin{bmatrix} -2 \\ 3 \\ 5 \end{bmatrix}$$

  **Exercise**: Verify the parallelogram law for vectors $\mathbf{u}$ and $\mathbf{v}$ in $\mathbb{R}^n$:

$$\|\mathbf{u} + \mathbf{v}\|^2 + \|\mathbf{u} - \mathbf{v}\|^2 = 2\|\mathbf{u}\|^2 + 2\|\mathbf{v}\|^2.$$

- Orthogonal and orthonormal

  **Exercise**:Determine if the sets of vectors is orthonormal. If it is only orthogonal, normalize them to an orthonormal set.

$$\begin{bmatrix} \sqrt{2} \\ 3 \\ 3 \end{bmatrix}, \begin{bmatrix} 6 \\ -\sqrt{2} \\ -\sqrt{2} \end{bmatrix}, \begin{bmatrix} 0 \\ -\sqrt{10} \\ -\sqrt{10} \end{bmatrix}$$

  **Exercise**:Find all the vectors in $\mathbb{R}^4$ that are orthogonal to

$$\mathbf{v} = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}.$$