HANDOUT 11

JIASU WANG

1. Determine if the following subsets W form a subspace of vector spaces V.
   (1) $V = P_2$, W is the set of polynomials of degree 2, i.e., $W = \{ax^2 + bx + c : a \neq 0\}$.
   No, 0 doesn't belong to W.
   (2) $V = P_2$, W is the set of polynomials that vanish at 1, i.e., $W = \{ax^2+bx+c : a+b+c = 0\}$.
   Yes, check three properties.

Remark: subspace = subset contains 0 + subset closed under addition and scalar multiplication.

Determinant: how to find the determinant of matrix?

- One method is doing row reduction:

$$\det(A) = \det(A \text{ after doing row i : row i + c row j})$$
$$\det(A) = (-1)\det(A \text{ after exchanging row i and row j})$$
$$\det(A) = \alpha \det\left(A \text{ after doing row i : row i } \times \frac{1}{\alpha}\right)$$

- $\det(AB) = \det(A)\det(B)$

2. Find the determinant of the following matrices
   (1) $\begin{bmatrix} 5 & 6 \\ 3 & -4 \end{bmatrix}$. $\Delta = -38$
   (2) $\begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{bmatrix}$. $\Delta = abc$
   (3) $\begin{bmatrix} 1 & 2 & 2 \\ -2 & 5 & -4 \\ 4 & 5 & -3 \end{bmatrix}$. $\Delta = -99$
   (4) $\begin{bmatrix} 1 & 4 & 2 \\ 3 & 5 & 1 \\ 2 & 1 & 6 \end{bmatrix}$. $\Delta = -49$